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Since this result is positive the thickness given by (2) corresponds to a minimum focal length, as might be anticipated from purely physical considerations.

It is thus clear that a reasonable interpretation of the statement of the problem leads to an analytical condition which is quite different from the given answer. The thickness of the lens cannot be considered negligible (as is often done in very elementary discussions of problems in geometrical optics) because the numerator of (2) cannot vanish when the upper sign is taken. In other words, t cannot be avoided in the final condition.

In conclusion, attention may be called to the fact that each of $\partial x/\partial r_1 = 0$, $\partial x/\partial r_2 = 0$, $\partial x/\partial \mu = 0$ leads to a result which bears no relation to the given answer and hence t is the only sensible independent variable.

349 (Calculus) [1913, 312]. Proposed by C. N. SCHMALL, New York City.

If $y = a \cos(\log x) + b \sin(\log x)$, eliminate the constants a and b and obtain the equation

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0.$$

SOLUTION BY T. E. MERGENDAHL, Tufts College.

Taking the derivative of y , as given in the problem, and multiplying the result by x , we have

$$x \frac{dy}{dx} = -a \sin \log x + b \cos \log x.$$

Applying the same process to both sides of this equation, we find

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} = -(a \cos \log x + b \sin \log x) = -y,$$

or

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0.$$

Also solved by W. W. BEMAN, T. M. BLAKSLEE, P. J. DA CUNHA, H. T. DAVIS, H. H. DOWNING, G. H. GRAVES, P. HANSEN, A. M. HARDING, W. L. MISER, A. PELLETIER, ELIJAH SWIFT, and H. S. UHLER.

385 (Calculus) [1915, 161; 1919, 73]. Proposed by H. B. PHILLIPS, Massachusetts Institute of Technology.

If $f(x)$ is continuous between a and x , show that

$$\lim_{n \rightarrow \infty} \frac{|n|}{(x-a)^n} \int_a^x \cdots \int_a^x f(x) dx^n = f(a).$$

T. H. GRONWALL, New York City, offers the following criticism and completion of the solution already published.

First, the passage to the limit for $n \rightarrow \infty$, which forms the last step (p. 74), requires justification (this is however easily done by using Taylor's theorem with remainder term). Second, the problem as proposed assumes only that $f(x)$ is continuous, so that the use of the derivatives of $f(x)$ is not permissible, since they may not exist. The following proof requires $f(x)$ to be bounded in the interval from a to x and to be continuous only at the point a , but not in the whole interval. It is readily shown (integration by parts and complete induction) that

$$\int_a^x \cdots \int_a^x f(x) dx^n = \frac{1}{(n-1)!} \int_a^x (x-\xi)^{n-1} f(\xi) d\xi^1$$

writing $\xi = a + t(x-a)$ and $f(\xi) = \varphi(t)$, the formula to be proved becomes

$$\lim_{n \rightarrow \infty} n \int_0^1 (1-t)^{n-1} \varphi(t) dt = \varphi(0)$$

¹ A proof of this relation may be found in Goursat-Hedrick's *Mathematical Analysis*, Vol. 2, pt. II, p. 36.—EDITORS.

(under the assumption that $\varphi(t)$ is continuous at $t = 0$), or what is the same

$$\lim_{n \rightarrow \infty} n \int_0^1 (1-t)^{n-1} [\varphi(t) - \varphi(0)] dt = 0.$$

Let $\epsilon > 0$ be as small as we please; $\varphi(t)$ being continuous at $t = 0$, it is then possible to choose a δ , $0 < \delta < 1$, and so small that $|\varphi(t) - \varphi(0)| < \epsilon/2$ for $0 \leq t \leq \delta$. Let M be greater than $|\varphi(t) - \varphi(0)|$ for $0 \leq t \leq 1$. Then, decomposing the integral into two parts and using the first mean value theorem,

$$\begin{aligned} \left| n \int_0^1 (1-t)^{n-1} [\varphi(t) - \varphi(0)] dt \right| &\leq \left| n \int_0^\delta \right| + \left| n \int_\delta^1 \right| \\ &\leq n \frac{\epsilon}{2} \int_0^\delta (1-t)^{n-1} dt + nM \int_\delta^1 (1-t)^{n-1} dt \\ &< \frac{\epsilon}{2} + M(1-\delta)^n. \end{aligned}$$

Now N may be chosen so large that $M(1-\delta)^n < \epsilon/2$ for $n > N$, so that finally

$$\left| n \int_0^1 (1-t)^{n-1} [\varphi(t) - \varphi(0)] dt \right| < \epsilon \text{ for } n > N,$$

which completes our proof.

192 (Number Theory) [1913, 196; 1919, 214]. Proposed by the late ARTEMAS MARTIN.

Find rational values of v , w , and x that will satisfy simultaneously the conditions

$$(m^2 + n^2)(v^2 + w^2 + x^2)^2 - 4m^2n^2v^2 + m^2n^2(m^2 + n^2) = 0,$$

$$(m^2 + n^2)(v^2 + w^2 + x^2)^2 - 4m^2n^2w^2 + m^2n^2(m^2 + n^2) = 0,$$

$$(m^2 + n^2)(v^2 + w^2 + x^2)^2 - 4m^2n^2x^2 + m^2n^2(m^2 + n^2) = 0,$$

m and n being known rational quantities.

DISCUSSION BY H. S. UHLER, Yale University.

The following analysis shows that the problem is impossible.

Any set of values of v , w , and x that fulfil the given conditions must also satisfy the sum of the three equations, that is, must satisfy

$$3(m^2 + n^2)(v^2 + w^2 + x^2)^2 - 4m^2n^2(v^2 + w^2 + x^2) + 3m^2n^2(m^2 + n^2) = 0.$$

From this equation, we have

$$v^2 + w^2 + x^2 = \frac{mn}{3(m^2 + n^2)} [2mn \pm i\sqrt{9m^4 + 14m^2n^2 + 9n^4}].$$

In general, the right hand member is irrational, and the sum of the squares of any number of rational numbers cannot have an irrational value; therefore, the problem is impossible.

We may proceed farther and find explicit expressions for v , w , and x . By inspection, or by forming the differences between the given conditions, taken in pairs, we see that $v^2 = w^2 = x^2$. Consequently,

$$x^2 = \frac{mn}{9(m^2 + n^2)} [2mn \pm i\sqrt{9m^4 + 14m^2n^2 + 9n^4}],$$

or

$$x = \pm \frac{1}{3} \sqrt{\frac{mn}{2(m^2 + n^2)}} \{ \sqrt{3(m^2 + n^2)} + 2mn \pm i \sqrt{3(m-n)^2 + 4mn} \}.$$

Obviously, these four roots are irrational, in general.